The Time Value of Money Guide

CFP Certification – Global Excellence in Financial Planning™
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Introduction

The opportunity to earn interest on money invested today makes money available now more valuable to us than the same amount of money not available in the future. This is the essence of what is frequently called the “the time value of money”. The principle of earning further interest on interest already received is referred to as compound interest.

This booklet aims to show you how to get the best use out of your financial calculator, using examples to build your understanding and test your knowledge. It also gives you some guided examples of how to apply this knowledge in everyday situations.

The service provided by the majority of financial advisers is investment and protection advice. This by definition is product orientated. If the client has a need for term assurance, a personal pension, an ISA, then the best product of that type is recommended.

Financial Planning is concerned with the identification and evaluation of the options faced by the client. The evaluation of options involves presenting the advantages and disadvantages of each. Part of this process of evaluation is the quantification of the financial outcomes - the likely financial costs or benefits of option one are ...... and of option two are ....... The quantification of financial consequences is an important input into the decision process by the client. Usually the client is in the best position to evaluate the non financial criteria that are to be taken into consideration. The client knows better than the planner how much risk they are prepared to take, how important it is to them to save money for the next generation, or what standard of life they aspire to in the years ahead and what they are prepared to sacrifice now to get it. But when it comes to putting numbers on the financial outcomes of the alternatives, the client will turn to a financial planner.

Quantifying the outcomes of the alternatives involves taking account of changes in values over time - the time value of money is at the very heart of professional Financial Planning.

This introduction aims to demonstrate to the unconvinced the centrality of the time value of money to personal Financial Planning.

NOTE - The examples that follow do not reflect current interest and inflation rates.
1. The Principles of Compound Interest

Assume that Mr & Mrs Andrews have £10,000 available for investment and that they place it on deposit in an account paying fixed interest of 10% per annum with the interest credited annually. At the end of the first year, their account will contain:

\[
\begin{align*}
\text{Capital of} & \quad £10,000 + \text{Interest of 10% x £10,000} \\
& \quad = £10,000 + 0.1 \times £10,000 \\
& \quad = £10,000 (1 + 0.1) \\
& \quad = £10,000 (1.1) \\
& \quad = £11,000
\end{align*}
\]

At the end of the second year, their account will contain:

\[
\begin{align*}
\text{Capital at end of year 1} + \text{Interest of 10% of capital at end of year 1} \\
& \quad = £11,000 + 0.1 \times £11,000 \\
& \quad = £10,000 (1.1) + 0.1 \times £10,000 (1.1) \\
& \quad = £10,000 (1.1) (1 + 0.1) \\
& \quad = £10,000 (1.1)(1.1) \\
& \quad = £10,000 (1.1)^2 \\
& \quad = £12,100
\end{align*}
\]

The interest that was credited to the account at the end of year 2 was £1,100 while that credited at the end of year 1 was only £1,000. The extra £100 at the end of year 2 was of course the 10% interest on the £1,000 interest credited to the account at the end of year 1. This growth of interest payable on previous interest is the principle of compound interest.

We can now see how much would be in Mr & Mrs Andrew’s account at the end of six years assuming that no interest or capital is withdrawn.

Table 1

<table>
<thead>
<tr>
<th>Number of years</th>
<th>Amount in the account</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>£10,000</td>
</tr>
<tr>
<td>1</td>
<td>£10,000 (1.1)</td>
</tr>
<tr>
<td>2</td>
<td>£10,000 (1.1) (1.1)   = £10,000 (1.1)^2</td>
</tr>
<tr>
<td>3</td>
<td>£10,000 (1.1)^2(1.1) = £10,000 (1.1)^3</td>
</tr>
<tr>
<td>4</td>
<td>£10,000 (1.1)^3(1.1) = £10,000 (1.1)^4</td>
</tr>
<tr>
<td>5</td>
<td>£10,000 (1.1)^4(1.1) = £10,000 (1.1)^5</td>
</tr>
<tr>
<td>6</td>
<td>£10,000 (1.1)^5(1.1) = £10,000 (1.1)^6</td>
</tr>
</tbody>
</table>

The following points can be observed in Table 1 above:

(1) The amount in the account at the end of the year is always equal to the amount in the account at the beginning of the year multiplied by one plus the rate of interest earned for the year.
(2) If the capital and accumulated interest in the account remain untouched and the rate of interest remains constant, the amount in the account is equal to the initial investment multiplied by one plus the annual rate of interest raised to the power of the number of complete years since the initial investment. This can be calculated using the formula below:

\[ FV_n = PV_0(1 + r)^n \]

Where:

- \( FV_n \) = Value of the account (future value) at the end of year \( n \).
- \( PV_0 \) = Initial investment (present value at outset or year 0).
- \( r \) = The rate of interest.
- \( n \) = The number of years.

**EXAMPLE**

The value of Mr & Mrs Andrew’s account at the end of 6 years would be:

\[
FV_6 = £10,000 \times (1.1)^6
\]

\[
= £10,000 \times 1.771561
\]

\[
= £17,715.61
\]

### 2. Solving Problems Using a Financial Calculator

Working out calculations such as the example above can be laborious. We would recommend that you obtain a financial calculator such as the HEWLETT-PACKARD 10B BUSINESS CALCULATOR which will be referred to in the text below. This is already programmed to perform financial calculations involving compound interest. Alternatively if you have a smart phone or iPad, you can download the 10bii Financial Calculator app, which works in the same way.

Every button on the calculator is designed to execute two functions. The first is provided on the button itself in white lettering. Thus by pressing each of the four buttons 7, 8, 9 and PV, you will have registered the figure of 789 as the present value. The second function is printed in orange above the button. To activate the second function, you must first press the orange coloured shift button followed by the required button. Thus to register that interest payments are calculated four times a year, you should press the button 4, followed by the orange shift button, followed by the P/YR button.

Please note that throughout the examples which follow, the words PRESS SHIFT are used to mean press the orange coloured shift button.
All of the figures in the examples in this booklet are rounded to 2 decimal places. To set your calculator to 2 decimal places:

<table>
<thead>
<tr>
<th>Step</th>
<th>Action</th>
<th>Display</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Press the C button to clear the display</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Press SHIFT DISP</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Press the figure 2</td>
<td>0.00</td>
</tr>
</tbody>
</table>

The five numerical variables we shall be using are in shown in white in the top row: N, I/YR, PV, PMT and FV.

N = the number of time periods  
I/YR = the interest rate  
PV = present value  
PMT = payment  
FV = future Value

After inputting the value of any four of these, the fifth can be computed by pressing the button of the fifth variable whose value is required. Sometimes you will have only three values and required to find the value of a fourth. The fifth variable must be given a value of zero before you attempt to calculate the value you require.

You should also note that cash outflows or investments made by the investor are preceded by a minus sign. Cash inflows or receipts by the investor are shown as positive amounts.

When using the Hewlett-Packard 10B, the first data the calculator requires you to provide is whether interest is to be calculated daily, monthly, quarterly, semi-annually, or annually. This is done by entering the appropriate number of times interest is credited per year (365, 12, 4, 2, or 1) followed by the SHIFT and the P/YR buttons.

The example of calculating the amount in Mr & Mrs Andrew’s account at the end of the sixth year can now be solved using the calculator by following the steps outlined below. Note that interest is to be credited to the account once a year.
3. Practice Examples

(1) Mr Baker invests a lump sum of £7,000 for four years at 12% payable annually. How much is in the account at the end of the four year period?

Answer: £11,014.64

(2) Mrs Clark invests a lump sum of £16,000 for 5.5 years at 12.5% payable annually. How much is in the account at the end of the 5.5 years? (The calculator will automatically allow for the interest in the last six months)

Answer: £30,581.51

4. Calculation of Present Values

Often clients may wish to accumulate a certain sum by a certain time and need to know how much they need to save over the time period to achieve their goal. For example, how much needs to be invested today to give £50,000 in six years time if the rate of interest is fixed at 10% and payable annually? The answer can be found as follows:

\[
\text{Since } FV_n = PV_0 (1+r)^n \\
\text{PV}_0 = \frac{FV_n}{(1+r)^n} \\
= \frac{£50,000}{(1.1)^6}
\]
This process is referred to as **DISCOUNTING**. The future sum of £50,000 is discounted at the rate of interest of 10% for six years to give the present value. Using the financial calculator to solve this equation we obtain £28,223.70. This means that if £28,223.70 is invested now at 10% for six years, at the end of the period the investment would be worth £50,000.

**Exercise**

Check that this is correct by performing the calculation in reverse. How much would £28,224 be worth if interest is added at 10% for six years?

The steps for this calculation on the Hewlett-Packard 10B are as follows:

<table>
<thead>
<tr>
<th>Step</th>
<th>Action</th>
<th>Display</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Enter: 1 (number of times per year interest is credited)</td>
<td>1.00</td>
</tr>
<tr>
<td>2</td>
<td>Press: SHIFT P/YR</td>
<td>1.00</td>
</tr>
<tr>
<td>3</td>
<td>Enter: 50,000 (future sum)</td>
<td>50,000</td>
</tr>
<tr>
<td>4</td>
<td>Press: FV</td>
<td>50,000.00</td>
</tr>
<tr>
<td>5</td>
<td>Enter: 10 (interest rate)</td>
<td>10.00</td>
</tr>
<tr>
<td>6</td>
<td>Press: I/YR</td>
<td>10.00</td>
</tr>
<tr>
<td>7</td>
<td>Enter: 6 (number of years)</td>
<td>6.00</td>
</tr>
<tr>
<td>8</td>
<td>Press: N</td>
<td>6.00</td>
</tr>
<tr>
<td>9</td>
<td>Enter: 0</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>Press: PMT</td>
<td>0.00</td>
</tr>
<tr>
<td>11</td>
<td>Press: PV</td>
<td>-28,223.70</td>
</tr>
</tbody>
</table>

**5. Practice Examples**

(1) Mr Danes requires £300,000 in seven years time to purchase the annuity he needs to provide him with his desired level of income in retirement. How much should he invest now if the expected investment return is 8% per annum credited annually?

Answer: £175,047.12

(2) Mrs Essex requires £420,000 in 9.75 years time to purchase her annuity. How much should she invest now if the expected investment return is 7.5% per annum credited annually?

Answer: £207,499.36
6. Changes in Interest Rates

The examples used so far have all assumed that the interest rate remains fixed throughout the period of the investment. As this may not be the assumption you wish to make, we shall now examine how a change of interest rate can be accommodated.

We have seen that if £10,000 is invested at 10% per annum for three years, with the interest credited annually, the value of the investment at the end of the 3 years would be:

\[ \£10,000 \times (1.1)^3 = \£10,000 \times (1.1)(1.1)(1.1) = \£13,310. \]

The original capital is multiplied by one plus the annual interest rate for each year of the investment. Thus if the interest rates for each year were to be 11%, 12% and 13%, the value of the investment at the end of the third year would be:

\[ \£10,000 \times (1.11)(1.12)(1.13) = \£14,048.16. \]

This formula can now be adapted to calculate present values instead of future values. For example, if the question in section 4 concerning how much needs to be invested now to produce £50,000 in 6 years time had assumed that interest rates were 11% for 2 years, 12% for 2 years and 13% for 2 years, the equation to be solved would have been:

Initial investment \( (PV_0) \) = \[ \frac{\£50,000}{(1.11)^2(1.12)^2(1.13)^2} \]

\[ = \frac{\£50,000}{1.9735} \]

\[ = \£25,335.70 \]

The denominator showing the amount by which the future value must be divided is obtained by multiplying together one plus the annual interest rate for each year of the investment.

7. Practice Examples

(1) Mr Fisher invests £150,000 for four years. The rates of return in each year are 11.5%, 12.5%, 9.0% and 11.5%. What is the value of his investment at the end of the four years?

Answer: £228,675.70

(2) Mrs Grey needs £175,000 in three years’ time. The expected rates of return in each year are 9.0%, 9.5% and 10.0%. How much money does she need to invest now?

Answer: £133,292.20
8. Frequency of Interest Payments

All the examples so far have assumed that interest is credited to the account annually. If interest is credited more frequently, then it means that interest can itself earn interest earlier and so enhance the total return. Interest rates may be quoted as either a nominal rate (which does not take account of the frequency of crediting interest) or the effective rate (which does take into account the frequency of crediting interest). In the UK, the effective rate of interest is frequently referred to as the compound annual return (CAR).

Examine the following example of an investment of £10,000 earning 12% nominal per annum, but where interest is credited at different time intervals.

<table>
<thead>
<tr>
<th>Frequency of interest</th>
<th>Amount at end of year</th>
<th>Compound Annual Return</th>
</tr>
</thead>
<tbody>
<tr>
<td>Annually</td>
<td>£11,200</td>
<td>12.00%</td>
</tr>
<tr>
<td>Semi-annually</td>
<td>£11,236</td>
<td>12.36%</td>
</tr>
<tr>
<td>Quarterly</td>
<td>£11,255</td>
<td>12.55%</td>
</tr>
<tr>
<td>Monthly</td>
<td>£11,268</td>
<td>12.68%</td>
</tr>
</tbody>
</table>

While the nominal rate in each case is 12%, the compound annual return is shown in the third column and reveals the effect on the total return of more frequent crediting of interest.

To find the compound annual return from the nominal interest rate, you should follow the steps outlined below. By way of example, let us calculate the compound annual return when the nominal rate is 12% and the interest is credited quarterly.

**Step** | **Action**
---|---
(1) | Divide the nominal annual rate of interest by the number of times interest is credited in the year.

Example: \(12\% ÷ 4 = 3\%\)

(2) | Express the percentage return as a decimal by dividing by one hundred, and then add one. Raise to the power of the number of interest payments credited per annum. Now subtract one and multiply by 100 to reconvert into a percentage.

Example: \((1.03^4 − 1) \times 100\)
\= (1.1255 − 1) \times 100\n\= 12.55\%\
There is another way in which you can calculate the effective interest rate using the Hewlett-Packard 10B which you might prefer.

(1) Enter the number of times interest is credited per year.
(2) Enter the nominal annual interest rate.
(3) Press the effective interest rate return.

The actions required are shown for the same example of a 12% nominal interest rate credited quarterly.

<table>
<thead>
<tr>
<th>Step</th>
<th>Action</th>
<th>Display</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Enter : 4</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>Press : SHIFT P/YR</td>
<td>4.00</td>
</tr>
<tr>
<td>4</td>
<td>Enter : 12</td>
<td>12</td>
</tr>
<tr>
<td>5</td>
<td>Press : SHIFT NOM%</td>
<td>12.00</td>
</tr>
<tr>
<td></td>
<td>Press : SHIFT EFF%</td>
<td>12.55</td>
</tr>
</tbody>
</table>

Now check that you agree with the compound annual returns of 12.36% and 12.68% shown in the table when interest is credited semi-annually and monthly. Use both methods of calculating the effective rate.

To use your Hewlett-Packard 10B calculator successfully, you must remember the following three points:

(1) The calculator assumes that time is divided into units of equal length. You fix this length when you enter the number of payments per year (SHIFT P/YR). Thus if you enter 1 payment per year, time is divided into annual intervals. If you enter four payments per year, time is divided into three monthly intervals.

(2) If you enter a number and press the N button, the calculator will assume that the number you have entered is the number of time intervals to which the interest rate should be applied. Thus if you specify 4 payments per year and then enter the figure 12 and press the N button, it will assume that the calculation should cover 12 quarters. If you mean 12 years and not 12 quarters, you should either:

   (i) Multiply the 12 years by four quarters in your head, enter 48 and press N.
   (ii) Enter 12 and press SHIFT xP/YR which will automatically multiply the 12 years by the number of payments per year and display a value of 48 for N.
(3) The interest rate you enter by pressing the I/YR button is assumed to be the annual nominal rate. If you have entered more than one payment per year, the calculator will divide the annual nominal rate by the number of time intervals in the year, and apply this rate to each time interval. Thus if you enter 4 payments per year giving quarterly time intervals, and an annual nominal interest rate of 12%, it will apply an interest rate of 12% / 4 = 3% to each quarter. This gives us an effective annual rate of interest of 12.55%.

You must therefore consider carefully whether you wish your calculation to assume an effective rate of 12.55% per annum or 12.00% per annum. If you mean that capital will grow by 12.55% per annum, then entering 12.00% as the interest rate per year with quarterly periods will provide the answer you require. If however you mean that the capital will only grow by 12.00%, then the annual nominal interest rate you should enter must be less than 12.00%! So how do you calculate the rate of interest to show as the nominal rate?

You should implement the following steps:

(1) Enter the number of payments per year.

(2) Enter the figure you intend to use as the assumed rate of growth of capital per year and press SHIFT EFF%. You have entered your assumed rate of growth as the effective rate of interest.

(3) Press SHIFT NOM% and the equivalent nominal rate is shown on the display. This takes into account the number of periods per year in which interest growth will be added to the capital value. The figure displayed has automatically been recorded as the interest rate per year. There is no need to press the I/YR button.
EXAMPLE

Assume Mr Hassan wishes to invest £10,000 for one year at the beginning of quarter one. At the start of quarters two, three and four, no further capital is added. At the end of the year, you know that the capital will have increased by 12.00% to £11,200, but you want to prove this using your calculator.

<table>
<thead>
<tr>
<th>Step</th>
<th>Action</th>
<th>Display</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Enter: 4 (divide the year into quarterly periods)</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>Press: SHIFT P/YR</td>
<td>4.00</td>
</tr>
<tr>
<td>3</td>
<td>Enter: 1 (the total number of periods – 1 year)</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>Press: SHIFT xP/YR</td>
<td>4.00</td>
</tr>
<tr>
<td>5</td>
<td>Enter: 10,000 (initial investment)</td>
<td>10,000</td>
</tr>
<tr>
<td>6</td>
<td>Press: +/-</td>
<td>-10,000</td>
</tr>
<tr>
<td>7</td>
<td>Press: PV</td>
<td>-10,000.00</td>
</tr>
<tr>
<td>8</td>
<td>Enter: 0 (to indicate no further investments)</td>
<td>0</td>
</tr>
<tr>
<td>9</td>
<td>Press: PMT</td>
<td>0.00</td>
</tr>
<tr>
<td>10</td>
<td>Enter: 12 (interest rate or capital growth rate)</td>
<td>12</td>
</tr>
<tr>
<td>11</td>
<td>Press: SHIFT EFF%</td>
<td>12.00</td>
</tr>
<tr>
<td>12</td>
<td>Press: SHIFT NOM%</td>
<td>11.49</td>
</tr>
<tr>
<td>13</td>
<td>Press: FV (to find capital value at end of year)</td>
<td>11,200.00</td>
</tr>
</tbody>
</table>

If in Step 10, you had entered 12 and then pressed the I/YR button, the FV calculation would have shown 11,255.09, an effective annual rate of return of 12.55%. Only by showing the rate of return you wanted as the effective rate and then calculating the nominal rate did we produce the answer you knew to be correct. Of course, had you shown the number of payments per year as equal to one, then the effective rate would have been equal to the nominal rate, and there would have been no need for the adjustments made in Step 10.

9. Regular Savings

A common situation encountered by the financial planner involving the use of compound interest is where the client wishes to make regular savings to achieve a desired lump sum target at a specified date in the future. Examples are where the client is saving for a specific expenditure such as meeting the education expenses of children or where the client needs to accumulate a lump sum by the time of retirement to purchase an annuity.
There are a number of variables involved in this type of calculation, and provided the size of all but one are known, then the unknown value can be easily calculated. The variables in question are:

1. The lump sum required (future value).
2. The assumed rate of return to be earned on investments.
3. The length of time available to accumulate the lump sum.
4. The frequency of the saving instalments.
5. Whether the first sum saved is immediate or at the end of the first interval between regular saving instalments.

**EXAMPLE**

Mr Inkstap needs £250,000 in 12 years time to purchase an annuity for his retirement. He is prepared to save money on a regular monthly basis. The effective annual rate of return on his investments is expected to be 12%. How much would the monthly saving need to be? The calculation of the required monthly saving can be broken down into the following stages:

<table>
<thead>
<tr>
<th>STEP</th>
<th>Action</th>
<th>Display</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Enter: 12 (divide the year into monthly periods)</td>
<td>12</td>
</tr>
<tr>
<td>2</td>
<td>Press: SHIFT P/YR</td>
<td>12.00</td>
</tr>
<tr>
<td>3</td>
<td>Enter: 250,000 (target future value)</td>
<td>250,000</td>
</tr>
<tr>
<td>4</td>
<td>Press: FV</td>
<td>250,000.00</td>
</tr>
<tr>
<td>5</td>
<td>Enter: 12 (number of time periods)</td>
<td>12</td>
</tr>
<tr>
<td>6</td>
<td>Press: SHIFT xP/YR</td>
<td>144.00</td>
</tr>
<tr>
<td>7</td>
<td>Enter: 0 (no initial lump sum)</td>
<td>0</td>
</tr>
<tr>
<td>8</td>
<td>Press: PV</td>
<td>0.00</td>
</tr>
<tr>
<td>9</td>
<td>Enter: 12 (annual effective rate of interest)</td>
<td>12</td>
</tr>
<tr>
<td>10</td>
<td>Press: SHIFT EFF%</td>
<td>12.00</td>
</tr>
<tr>
<td>11</td>
<td>Press: SHIFT NOM%</td>
<td>11.39</td>
</tr>
</tbody>
</table>

Finding the value of the required monthly payment:

- If the first payment is to be immediate, ensure that **BEGIN** is displayed. If not, press **SHIFT BEG/END**. Then press the **PMT** button.
- If the first payment is to be at the end of the first month, ensure that **BEGIN** is not displayed. If it is, press **SHIFT BEG/END** to remove it.

The two results are:

1. Immediate payment: **£811.44**
2. First payment at the end of the month: **£819.14**

**N.B.** The payment will show a negative sign to represent an outflow of cash.
10. Practice Examples

(1) Ms Jones says that she can afford to save no more than £120 per month. What sum will she accumulate in five years if she starts saving at the end of the month and the effective annual rate of return on her investment is 10%?

Answer: £9,187.35

(N.B. The payment of £120 should be preceded by a negative sign to represent an outflow)

(2) What difference would it make if:

(i) the first payment is at the start of the month?
(ii) the nominal rate of return and not the effective rate of return is equal to 10%?

Answers: (i) £9,260.61
(ii) Beginning: £9,369.89
    End: £9,292.45

11. A Lump Sum Investment Plus Regular Savings

A variation upon the situation described in section 9 is where the client already has a lump sum to invest and wishes to know how much needs to be invested on a regular basis to accumulate a given sum by a specified date.

EXAMPLE

Mr King has £40,000 at present and wishes to accumulate £250,000 in 12 years by a regular monthly saving scheme. If we assume that the expected effective rate of return on the investment is 12% per annum, how much must be invested each month if the first payment is immediate?

The calculation is the same as in the example shown in section 9, but instead of setting PV = 0, you should enter 40,000, press the +/- button, and then press the PV button.

The amount that must be invested per month assuming the first payment is immediate is reduced from £811.44 to £305.62.
12. Practice Example

(1) Mr Long has £65,000 already saved towards the pension annuity he wishes to purchase in nine years time. You have calculated that he will need £320,000 and have agreed that the expected effective rate of return on investment is 12% per annum. Assuming he makes annual contributions and the first is immediate, what will the annual investment need to be?

Answer: £8444.75

13. Calculating the Rate of Return on an Investment

Of the 5 numerical variables N, I/YR, PV, PMT and FV, any one can be calculated provided the values of the other four are known. In sections 10 and 12, we have calculated the values of FV and PMT. Another situation often met by a financial planner involves calculating the rate of return on investment.

EXAMPLE

A life company’s ten year endowment policy with a monthly premium of £100 produced a maturity value of £19,108. The implied return on investment can be calculated as follows:

<table>
<thead>
<tr>
<th>Step</th>
<th>Action</th>
<th>Display</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Enter: 12 (payments per year)</td>
<td>12</td>
</tr>
<tr>
<td>2</td>
<td>Press: SHIFT P/YR</td>
<td>12.00</td>
</tr>
<tr>
<td>3</td>
<td>Check that BEGIN is shown in the display as the first payment is immediate.</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Enter: 19,108 (future value)</td>
<td>19,108</td>
</tr>
<tr>
<td>5</td>
<td>Press: FV</td>
<td>19,108.00</td>
</tr>
<tr>
<td>6</td>
<td>Enter: 100 (monthly premium)</td>
<td>100</td>
</tr>
<tr>
<td>7</td>
<td>Press: +/-</td>
<td>-100</td>
</tr>
<tr>
<td>8</td>
<td>Press: PMT</td>
<td>-100.00</td>
</tr>
<tr>
<td>9</td>
<td>Enter: 10 (number of years)</td>
<td>10</td>
</tr>
<tr>
<td>9</td>
<td>Press: SHIFT xP/YR</td>
<td>120.00</td>
</tr>
<tr>
<td>10</td>
<td>Enter: 0 (present value)</td>
<td>0</td>
</tr>
<tr>
<td>11</td>
<td>Press: PV</td>
<td>0.00</td>
</tr>
<tr>
<td>12</td>
<td>Press: I/YR</td>
<td>8.65</td>
</tr>
</tbody>
</table>

The nominal investment rate of return is calculated by pressing the I/YR button to be 8.65%. The effective annual rate given monthly compounding can be calculated by pressing SHIFT EFF% to be 9.00%.
14. Practice Example

(1) Mrs Miller says that she can afford to save £250 per month for the next nine years. She would like to accumulate £50,000 in that time. What effective rate of return would her investments need to yield to achieve this goal if (i) she started saving immediately and (ii) she started saving at the end of the month? (N.B. the payment of £250 per month should be preceded by a negative sign)

Answers:
(i) 13.24% per annum
(ii) 13.46% per annum

15. Escalating Savings Plans

While you were doing the calculations in 10 - 14, you were probably wondering how to handle the questions if the amount invested each year were to increase rather than remain constant. Let us use an example to demonstrate how annual increments in contributions can be included in the calculation:

EXAMPLE
Mr North needs £75,000 in six years time to purchase an annuity for his retirement. He is prepared to save money on a regular monthly basis and to increase these savings by 10% per annum. The effective rate of return on his investments is expected to be 12%. What would be the required amount of the first month’s investment?

Table 2 overleaf sets the scene. There are six periods of twelve months until the £75,000 is required. Let us assume that if Mr North was going to make six annual payments with the first made immediately, the amount required for the first payment would be £X. In twelve months time, a second investment of £X(1.1) would be made and then every 12 months, a further investment 10% higher than the previous one. The sixth and final investment would be made in 60 months and would equal £X(1.1)^5.

If Mr North did not make his first investment until the end of the first year, he would need to invest a larger sum £A to compensate for the fact that there are now only 60 months left in which to accumulate the required £75,000. After 24 months, Mr Client would make his second investment of £A(1.1) and after 72 months, his sixth and final investment of £A(1.1)^5.
Table 2 - Timing of annual savings to accumulate a lump sum

<table>
<thead>
<tr>
<th>Months</th>
<th>A</th>
<th>A(1.1)</th>
<th>A(1.1)^2</th>
<th>A(1.1)^3</th>
<th>A(1.1)^4</th>
<th>A(1.1)^5</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>12</td>
<td>24</td>
<td>36</td>
<td>48</td>
<td>60</td>
<td>72</td>
</tr>
</tbody>
</table>

Since the values of £X and £A represent the first year’s saving required if the payment is at the beginning or end of the year, their average would give the saving required if paid in the middle of the year. Dividing this average by 12 gives the monthly payment required during the first year.

The steps you need to take to calculate the values of £X and £A are listed below. There is no explanation as to why you should take these steps; this follows in section 17.

**Calculation of the value of £X, the amount of the first annual investment if made at the start of the first year.**

1. Enter 1 payment per year.
2. Check that BEGIN is displayed.
3. Divide the required future value by one plus the annual rate of increase in the amount invested, raised to the power of the number of years of investment growth available to the first payment. Press the FV button to enter this value.

In this example, the required future value is £75,000, the annual rate of increase in saving is 10 per cent and the number of years of investment growth for the first premium is 6 years.

Thus we have:

\[ \text{FV} = \frac{75,000}{(1.1)^6} = 42,335.54 \]

4. Calculate the annual investment rate of return to be used. This is one plus the annual rate of investment growth divided by one plus the rate of increase in the amount invested; subtract one, and multiply by one hundred to convert into a percentage.

Press the I/YR button to enter this value. [NB Since the payments are annual, the effective rate and the nominal rate are the same].

In this example, the annual investment return is 12% and the annual rate of increase in contributions is 10%. Thus the interest rate per year to be used in the calculation is:

\[ \frac{1.12 - 1 \times 100}{1.10} = 1.82 \]
The display on the calculator rounds to 2 decimal places but actually performs the calculation to 12 significant figures. Please note that all the calculations in the rest of this chapter assume that the interest rate is calculated by undertaking all the necessary steps shown above, and not by inserting the interest rate rounded to 2 decimal places.

(5) Enter the number of years until the future value is required and press the N Button.

(6) Press the zero button followed by the PV button to show that present value does not enter this calculation.

(7) Press the PMT button to compute the required first annual payment (£6621.55)

**Calculation of the value of £A, the amount of the first investment if made at the end of the first year.**

To obtain the value of £A, multiply the value of £X by one plus the assumed annual rate of investment growth.

In this example:

\[
£6,621.55 \times (1.12) = £7,416.14
\]

**Calculation of the monthly saving required during the first year.**

To obtain the first year’s monthly saving, we take the average of the annual premiums required if the first payment is at the beginning of the year and if the first payment is at the end of the year. This will give the annual saving required if paid in the middle of the year or if spread evenly throughout the year. The monthly saving required can therefore be found by dividing this average by 12. In this example, we obtain:

\[
\frac{£6,621.55 + £7,416.14}{2 \times 12} = £584.90
\]

The size of this monthly investment must be increased annually by 10% so that by the end of the sixth year, the sum accumulated will equal £75,000.
16. Practice Example

(1) Mrs Orwell needs £10,000 in three years time. She intends to increase the amount saved by 8% per annum and expects the rate of return on her investments to equal 10% per annum. How much does she need to save per month in the first year?

Answer: (£2550.47 + £2805.52) ÷ 24 = £223.17 per month

17. Calculations for the Annual Escalation of Savings

This section develops section 15, giving justification for the rules presented there for calculating a monthly saving contribution which increases annually.

Calculation of the value of £X, the amount of the first investment if made at the commencement of the first year.
Table 3 below shows the future values of the six annual investments at the end of the six years assuming that the first annual investment of £X is immediate.

Column 2 shows the amount of new saving at the beginning of each year assuming an annual rate of increase of 10%.

Table 3 Mr North’s Required Annual Saving (First Investment Immediate)

<table>
<thead>
<tr>
<th>(1) NUMBER OF YEARS TO INVESTMENT</th>
<th>(2) AMOUNT INVESTED</th>
<th>(3) INVESTMENT GROWTH</th>
<th>(4) FUTURE VALUE ££</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>X</td>
<td>(1.12)^6</td>
<td>X (1.12)^6</td>
</tr>
<tr>
<td>1</td>
<td>X(1.1)</td>
<td>(1.12)^5</td>
<td>X (1.1) (1.12)^5</td>
</tr>
<tr>
<td>2</td>
<td>X(1.1)^2</td>
<td>(1.12)^4</td>
<td>X (1.1)^2 (1.12)^4</td>
</tr>
<tr>
<td>3</td>
<td>X(1.1)^3</td>
<td>(1.12)^3</td>
<td>X (1.1)^3 (1.12)^3</td>
</tr>
<tr>
<td>4</td>
<td>X(1.1)^4</td>
<td>(1.12)^2</td>
<td>X (1.1)^4 (1.12)^2</td>
</tr>
<tr>
<td>5</td>
<td>X(1.1)^5</td>
<td>(1.12)</td>
<td>X (1.1)^5 (1.12)</td>
</tr>
</tbody>
</table>

Column 3 shows that the first year’s investment of £X has six years to grow at 12%, and each year’s investment thereafter has one less year to grow at 12%. The final investment in 5 years time, the commencement of the sixth year, will have one year to grow at 12%. The total future values of all these investments are shown in column 4 and must equal £75,000 at the end of the sixth year.
To find the value of £X, we need to rearrange the data in Table 3 in order to get it into a format so that we can use the calculator. We shall start by dividing each future value in column 4 by one plus the annual rate of increase in the amount invested raised to the power of the number of years of investment growth available for the first payment. In the example in Table 3, this means dividing each entry in the final column by (1.1)^6. Thus we obtain:

\[
\frac{X (1.1)^6}{(1.1)^6} + \frac{X (1.1)^5}{(1.1)^5} + \ldots + \frac{X (1.1)^2}{(1.1)^2} + \frac{X (1.1)}{(1.1)} = \frac{75,000}{(1.1)^6} = \£42,335.54
\]

This is the formula for a regular savings plan consisting of six annual investments of £X and a future value in six years time of £42,335.54. The investment rate of return is equal to:

\[
\frac{1.12 - 1}{1.1} \times 100 = 1.82\%
\]

The first payment is immediate because the first £X in the equation is multiplied by one plus the rate of investment return raised to the power of the number of years available for its investment growth - in this case six years.

We can now find the value of £X using the calculator as follows:

<table>
<thead>
<tr>
<th>Step</th>
<th>Action</th>
<th>Display</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Enter: 1 (number of payments per year)</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>Press: SHIFT P/YR</td>
<td>1.00</td>
</tr>
<tr>
<td>3</td>
<td>Check BEGIN is shown on the display</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Enter: the other known values:</td>
<td></td>
</tr>
<tr>
<td></td>
<td>I/YR</td>
<td>1.82</td>
</tr>
<tr>
<td></td>
<td>FV</td>
<td>42,335.54</td>
</tr>
<tr>
<td></td>
<td>PV</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>N</td>
<td>6.00</td>
</tr>
<tr>
<td>5</td>
<td>Press PMT</td>
<td>-6621.55</td>
</tr>
</tbody>
</table>

Thus a payment of £6,621.55 made immediately, and increasing by 10% for five further years would produce £75,000 at the end of 6 years if the rate of investment growth were 12%.

**Calculation of the value of £LA, the amount of the first investment if made at the end of the first year.**

Now we have to calculate the size of the first annual investment if it were not made until the end of the first year. Let the size of the first annual investment be £LA, and the next five investments increase by 10% per annum so that the sixth investment at the end of the sixth year would equal £A(1.1)^5. The size of these annual investments, their growth and final value are shown in Table 4 below.
Table 4  Mr North’s Required Annual Saving  
(First Investment At The End of Year 1)

<table>
<thead>
<tr>
<th>(1) NUMBER OF YEARS TO INVESTMENT</th>
<th>(2) AMOUNT INVESTED £</th>
<th>(3) INVESTMENT GROWTH £</th>
<th>(4) FUTURE VALUE £</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>A</td>
<td>$(1.12)^5$</td>
<td>$X (1.12)^5$</td>
</tr>
<tr>
<td>2</td>
<td>A$(1.1)$</td>
<td>$(1.12)^4$</td>
<td>$X (1.1) (1.12)^4$</td>
</tr>
<tr>
<td>3</td>
<td>A$(1.1)^2$</td>
<td>$(1.12)^3$</td>
<td>$X (1.1)^2 (1.12)^3$</td>
</tr>
<tr>
<td>4</td>
<td>A$(1.1)^3$</td>
<td>$(1.12)^2$</td>
<td>$X (1.1)^3 (1.12)^2$</td>
</tr>
<tr>
<td>5</td>
<td>A$(1.1)^4$</td>
<td>$(1.12)^1$</td>
<td>$X (1.1)^4 (1.12)^1$</td>
</tr>
<tr>
<td>6</td>
<td>A$(1.1)^5$</td>
<td></td>
<td>$X (1.1)^5$</td>
</tr>
<tr>
<td><strong>TOTAL</strong></td>
<td></td>
<td></td>
<td><strong>75,000</strong></td>
</tr>
</tbody>
</table>

To find the value of £A, we shall proceed in the same way as for the previous calculation. We divide each entry in the final column by $(1.1)^5$ - one plus the annual rate of increase in the amount invested raised to the power of the number of years of investment growth available for the first payment. This gives us:

$$A \frac{(1.12)^5}{(1.1)^5} + A \frac{(1.12)^4}{(1.1)^4} + \ldots + A \frac{(1.12)^1}{(1.1)^1} + A = \frac{75,000}{(1.1)^5} = 46,569.10$$

This is the formula for a regular savings plan consisting of 6 annual investments of £A and a future value in six years time of £46,569.10. The investment rate of return is the same as in the previous calculation and is equal to:

$$\frac{1.12 - 1 \times 100}{1.1} = 1.82\%$$

We know that the first payment is at the end of the first year and this is demonstrated by the fact that the first investment of £A is multiplied by one plus the rate of investment return raised to the power of the number of years available for its investment growth – in this case only 5 out of the 6 years.
We can now find the value of £A using the calculator as follows:

<table>
<thead>
<tr>
<th>Step</th>
<th>Action</th>
<th>Display</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Enter: 1 (number of payments per year)</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>Press: SHIFT P/YR</td>
<td>1.00</td>
</tr>
<tr>
<td>3</td>
<td>Ensure that BEGIN is not shown on the display.</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Enter the other known values:</td>
<td></td>
</tr>
<tr>
<td></td>
<td>I/YR</td>
<td>1.82</td>
</tr>
<tr>
<td></td>
<td>FV</td>
<td>46549.10</td>
</tr>
<tr>
<td></td>
<td>PV</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>N</td>
<td>6.00</td>
</tr>
<tr>
<td>5</td>
<td>Press: PMT</td>
<td>-7416.14</td>
</tr>
</tbody>
</table>

Thus the annual investment needs to be £6621.55 if the first investment is made *at the beginning of the first year* and £7416.14 if the first investment is made *at the end of the first year*.

Note how the difference between the two required annual investments is 12% due to the fact that the cost of delaying the investments from the beginning to the end of the year is 12% investment growth.

\[ £6621.55 \times (1.12) = £7416.14 \]

Compare column 3 in Table 3 with the same in Table 4 and note how each row’s savings in Table 4 receives one year less of investment growth. Hence the quick way to find the value of £A is to multiply £X by one plus the assumed rate of investment growth.

**Calculation of the monthly saving required during the first year.**

To obtain the first year’s monthly saving, we take the average of the annual premiums required if the first payment is at the beginning of the year and if the first payment is at the end of the year. This will give the annual saving required if paid in the middle of the year or if spread evenly throughout the year. The monthly saving required can therefore be found by dividing this average by 12.

The monthly saving required in the first year will therefore be approximately equal to:

\[ (£6621.55 + £7416.14) \div (2 \times 12) = £584.90 \text{ per month} \]

The size of this monthly investment must be increased annually by 10%, and assuming the investment rate of return of 12.00% is achieved, at the end of the sixth year, the sum accumulated will be £75,000.
18. A Lump Sum Investment, Plus Escalating Regular Savings

If Mr North, who needs £75,000 in six years time to purchase an annuity for retirement, already has £25,000 in his pension plan available to contribute towards this sum, the amount of monthly investment required will be reduced compared with the figures worked out in Section 17. Assume the effective annual rate of investment return remains at 12% and the annual rate of escalation of monthly saving remains at 10%. The new calculation of the required monthly investment is undertaken as follows:

(1) Calculate the value of the £25,000 currently available in six years time.

\[ £25,000 (1.12)^6 = £25,000 (1.97) = £49,345.57 \]

Thus the sum that needs to be accumulated by regular savings is £75,000 - £49,345.57 = £25,654.43

(2) Calculate the required annual savings in the same way as in section 17 except that the required future value is now £25,654.43 and not £75,000.

Assuming immediate investment, the first year’s saving required would be £2,264.96. If the first annual saving does not commence until the end of the first year, the amount required would be £2,536.76. Thus the required first year’s monthly investments would be approximately:

\[ (£2,264.96 + £2,536.76) ÷ 24 = £200.07 \]

N.B. Unlike in section 11 covering a lump sum investment combined with the regular level saving, you must not enter the existing capital as equal to the present value. You must split the calculation into the two parts shown above.

19. Practice Example

In section 16, Mrs Orwell needs £10,000 in three years’ time. She intends to increase the amount saved by 8% per annum and expects the rate of return on her investments to be equal to 10% per annum. What difference would it make to the amount she needs to save per month in the first year if she already has £2,500 to invest?

Answer: \( (£1701.80 + £1871.98) ÷ 24 = £148.91 \) per month compared with £223.17 per month.
20. Calculating the Required Rate of Return

A variation on the Escalating Saving Plan in section 15 is where the initial payment and the annual rate of growth of this payment are known, and we wish to calculate the investment rate of return required to produce a specified future sum.

EXAMPLE

Mr Quest wishes to invest £1,000 per annum with an annual rate of increase of 5% per annum. The first investment will be immediate. The sum of money needed in three years from now is £5,000. What is the required rate of return on investment?

The steps you need to take to calculate the answer are listed below without any explanation regarding why they will produce the answer you require. The following section will provide justification.

<table>
<thead>
<tr>
<th>Step</th>
<th>Action</th>
<th>Display</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Enter: 1 (number of payments per year)</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>Press: SHIFT P/YR</td>
<td>1.00</td>
</tr>
<tr>
<td>3</td>
<td>Check that BEGIN is shown in the display</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Divide the required future value by one plus the rate of annual increase in payment raised to the power of the number of years of investment. In this case, we divide £5,000 by (1.05)^3 to obtain £4,319.19</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Enter: 4319.19 from step 4</td>
<td>4319.19</td>
</tr>
<tr>
<td>6</td>
<td>Press: FV</td>
<td>4319.19</td>
</tr>
<tr>
<td>7</td>
<td>Enter the remaining known values: PMT</td>
<td>-1,000.00</td>
</tr>
<tr>
<td></td>
<td>N</td>
<td>3.00</td>
</tr>
<tr>
<td></td>
<td>PV</td>
<td>0.00</td>
</tr>
<tr>
<td>8</td>
<td>Press: I/YR (interest rate)</td>
<td>19.37</td>
</tr>
</tbody>
</table>

To find the required rate of return:

9 Divide the interest rate by 100 to convert from a percentage to a decimal and add 1.

\[(19.37 \div 100) + 1 = 1.1937\]

10 Multiply by one plus the rate of annual increase in saving.

\[1.1937 \times 1.05 = 1.25\]

11 Subtract 1, and multiply by 100 to convert into a percentage.

\[(1.25 - 1) \times 100 = 25.33\%\]
CALCULATIONS FOR THE REQUIRED RATE OF RETURN

The problem examined in section 20 has been set out in Table 5 below:

Table 5  Mr Quest: Required Rate of Return on Investment

<table>
<thead>
<tr>
<th>Number of years to investment</th>
<th>Contribution at start of year growing at 5% p.a.</th>
<th>Investment growth at Y% p.a.</th>
<th>Final Value £</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1000 (1.05)^3</td>
<td>(1+Y)^3</td>
<td>1000 (1+Y)^3</td>
</tr>
<tr>
<td>1</td>
<td>1000 (1.05)</td>
<td>(1+Y)^2</td>
<td>1000 (1.05) (1+Y)^2</td>
</tr>
<tr>
<td>2</td>
<td>1000 (1.05)^2</td>
<td>(1+Y)</td>
<td>1000 (1.05)^2 (1+Y)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>5,000</td>
</tr>
</tbody>
</table>

To find the value of Y, we start by dividing each entry in the final column by one plus the rate of annual increase in payment raised to the power of the number of years of investment growth available for the first investment. In this case, we divide by (1.05)^3 to obtain:

\[
\frac{1000 (1+Y)^3}{(1.05)^3} + \frac{1000 (1+Y)^2}{(1.05)^2} + \frac{1000 (1+Y)}{(1.05)} = \frac{5000}{(1.05)^3} = 4319.19
\]

This equation is the same as for a regular savings plan consisting of 3 annual investments of £1,000 and a future value in 3 years’ time of £4,319.19. The first payment is immediate because it is multiplied by one plus the rate of investment return raised to the power of the number of years available for its investment growth – in this case 3 years.

The investment rate of return is equal to:

\[
\frac{(1+Y) - 1}{1.05} \times 100
\]

Thus to find the value of Y, we must first fine the value of the interest rate per year which satisfies this equation. The calculation undertaken earlier in section 20 has shown to be 19.37%. Thus we have:

\[
\frac{1+Y}{1.05} = 1.1937
\]

Thus to find the value of Y, we must first fine the value of the interest rate per year which satisfies this equation. The calculation undertaken earlier in section 20 has shown to be 19.37%. Thus we have:

\[
\frac{1+Y}{1.05} = 1.1937
\]

\[
Y = 25.33\%
\]
21. Practice Example
A quotation from an insurance company states that a premium of £1,200 per annum increasing at 5% per annum will produce an investment fund of £24,150 in 10 years’ time. What is the assumed rate of return on the investment?

Answer: 9.00%

22. The Effect of Inflation on Regular Savings

22.1 Introduction
The problem with the examples of regular savings that we have examined so far is that they ignore the effects of inflation. A fixed income that is just sufficient to sustain a satisfactory standard of living today will not be sufficient in 5 years’ time.

This is an appropriate point at which to mention the “Rule of 72”. This rule states quite simply that if you divide 72 by the rate of interest, you obtain the number of years that it takes for an investment earning that rate to double in value. Similarly, with inflation at 8% per annum, it takes 9 years for the cost of living to double or the value of money to halve.

Clients rely upon their financial planners to incorporate protection against the effects of inflation in their recommendations. Of course, financial planners have no special insight into what the future rates of inflation will be in the same way that they have no accurate insight into what the future rate of investment returns will be. We must, however, make reasonable assumptions and incorporate them in our calculations and hence recommendations.

There are two alternative ways in which we can take account of the effects of inflation in our calculations. What we shall see is that the application of either of these methods results in the same calculation, and hence it does not matter in principle which method is used.

22.2 Adjustment for inflation: the use of constant prices
The first method of allowing for the effects of inflation is to undertake the analysis of the client’s affairs in terms of constant prices. Thus the lump sum which the client requires to purchase an annuity on retirement at the appropriate date in the future will be the same as if it were required today. However, we must recognise that if the cash flows are not adjusted for inflation because they are expressed in constant value or real terms, the same should apply to the rate of return on investments. It would be helpful here to distinguish clearly between the concepts of the nominal rate of return and the real rate of return.

The NOMINAL RATE OF RETURN is that earned in terms of current prices which increase with inflation.

The REAL RATE OF RETURN is that measured in terms of constant prices after excluding the effects of inflation.
A simple, but slightly inaccurate way to adjust the nominal rate for the effect of inflation is to subtract the rate of inflation from the nominal value. Thus a nominal rate of 9% at a time when inflation is measured at 5% gives a real rate of return of 4%.

A more accurate calculation of the real rate of return involves discounting the nominal rate of return by the rate of inflation using the formula:

\[
1 + r = \frac{1+n}{1+i}
\]

\[
r = \frac{1+n}{1+i} - 1
\]

Where

- \( r \) = real rate of return
- \( n \) = nominal rate of return
- \( i \) = rate of inflation

Thus in the example used above, the real rate of return can be calculated to be:

\[
r = \frac{1.09}{1.05} - 1 \times 100 = 3.81\%
\]

**Example – Mr Rush**

Let us examine an example using constant prices and a real rate of return. Assume that you have found from quotations provided by life companies that if Mr Rush were to retire today, he would need £100,000 cash to purchase the index linked pension annuity that would meet his aspirations concerning his future standard of living. We will assume that Mr Rush has six years in which to achieve this level of savings and that he has agreed to pay six annual instalments, starting now, into a pension plan to meet this objective. After discussion with Mr Rush, you have agreed with him to use a predicted rate of inflation over this period of 5% per annum and an annual tax free rate of inflation over this period of 5% per annum and an annual tax free rate of return on pension investment funds of 9%. These figures give a real rate of return on investment of 3.81% when shown to 2 decimal places. Now we need to calculate the annual investment required in constant prices given that there are six instalments and the future value required is £100,000. The calculation can be performed as follows:

<table>
<thead>
<tr>
<th>Step</th>
<th>Action</th>
<th>Display</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Enter: 1 (number of payments per year)</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>Press: SHIFT P/YR</td>
<td>1.00</td>
</tr>
<tr>
<td>3</td>
<td>Check that BEGIN is shown on the display</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Enter the following known values:</td>
<td></td>
</tr>
<tr>
<td></td>
<td>FV</td>
<td>100,000</td>
</tr>
<tr>
<td></td>
<td>N</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>I/YR</td>
<td>3.81</td>
</tr>
<tr>
<td></td>
<td>PV</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>Press: PMT</td>
<td>-14,592.63</td>
</tr>
</tbody>
</table>
We have calculated that the first investment by Mr Rush must be £14,592.63. Under conditions of constant prices, each of the five subsequent investments will also need to equal £14,592.63.

However, since we assumed inflation was 5% per annum, the annual investments must increase by 5% per annum and can therefore be easily calculated to equal £15,322.26, £16,088.37, £16,892.79, £17,737.43 and £18,624.30.

22.3 Adjustment for inflation: the use of actual prices
The alternative method is to work with the predicted actual cash flows which we have calculated using our predicted inflation rate. Applying this method to the example used in the previous section, we shall need to carry out the following steps:

(1) Calculate the sum required in six years time to purchase the desired annuity given that inflation is 5% per annum. Since the amount required today would have been £100,000, the amount required in six years’ time would be £100,000 (1.05)^6 = £134,010.

(2) Calculate the first of the six instalments to be invested now. This amount will be increased by 5% per annum for each of the subsequent lump sum investments to offset the effect of inflation and maintain the real value of contributions.

Let the letter X denote the amount that must be invested now as the first instalment. Each year’s contribution is shown in the second column in Table 6 below.

### Table 6
Mr Rush: Amounts to be Invested and Their Future Values at Retirement in Six Years Time

<table>
<thead>
<tr>
<th>NUMERO OF YEARS TO INVESTMENT</th>
<th>AMOUNT INVESTED</th>
<th>FUTURE VALUE IN YEAR 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>X</td>
<td>X(1.09)^6</td>
</tr>
<tr>
<td>1</td>
<td>X(1.05)</td>
<td>X(1.05)(1.09)^4</td>
</tr>
<tr>
<td>2</td>
<td>X(1.05)^2</td>
<td>X(1.05)^2(1.09)^4</td>
</tr>
<tr>
<td>3</td>
<td>X(1.05)^3</td>
<td>X(1.05)^3(1.09)^3</td>
</tr>
<tr>
<td>4</td>
<td>X(1.05)^4</td>
<td>X(1.05)^4(1.09)^4</td>
</tr>
<tr>
<td>5</td>
<td>X(1.05)^5</td>
<td>X(1.05)^5(1.09)^5</td>
</tr>
<tr>
<td>TOTAL</td>
<td></td>
<td>134,010</td>
</tr>
</tbody>
</table>

The third column shows the future value at retirement of each year’s investment assuming a constant rate of return on investments of 9%. Thus the first instalment can experience six years of compound growth at 9%, while the sixth instalment at the commencement of the sixth year will only have one year of growth.

The total value of the third column must equal £100,000 (1.05)^6 = £134,010.
To find the value of $X$, we follow the method shown in sections 17 and 20. We divide the future value of each year’s investment in year six and their total value by $(1.05)^6$, one plus the rate of annual increase in the amount invested raised to the power of the number of years investment growth available for the first investment. This gives:

$$X(1.09)^6 + X(1.09)^5 + X(1.09)^4 + X(1.09)^3 + X(1.09)^2 + X(1.09) = 100,000$$

This is the equation for a savings plan of £X per annum growing at $(1.09 ÷ 1.05 - 1) \times 100\%$ per annum. This is the same rate of 3.81% as calculated in section 22.2. Solving for the value of $X$, the first payment, requires the same calculation as shown in the previous section:

<table>
<thead>
<tr>
<th>Step</th>
<th>Action</th>
<th>Display</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Enter: 1 (number of payments per year)</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>Press: SHIFT P/YR</td>
<td>1.00</td>
</tr>
<tr>
<td>3</td>
<td>Check that BEGIN is shown on the display</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Enter the following known values:</td>
<td></td>
</tr>
<tr>
<td></td>
<td>FV</td>
<td>100,000</td>
</tr>
<tr>
<td></td>
<td>N</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>I/YR</td>
<td>3.81</td>
</tr>
<tr>
<td></td>
<td>PV</td>
<td>0.00</td>
</tr>
<tr>
<td>5</td>
<td>Press: PMT</td>
<td>-14,592.63</td>
</tr>
</tbody>
</table>

We have reached our intended destination! We have shown that calculating the required annual savings using constant prices and a real rate of return will result in the same calculation and hence yield the same figures as using current prices and the nominal rate of return.

**A More Complicated Example – Mr Stone**

It has been the norm for national average earnings to rise faster than the rate of inflation. This may result in a client who is saving for retirement increasing the annual amount saved at the faster rate of increase in earnings rather than the rate of price inflation. How would we calculate the required annual saving to obtain a specified lump sum in this situation?

The example in the previous two sections dealt with a client who requires £100,000 measured in terms of today’s prices in six years time to purchase an index linked annuity. The assumed rate of inflation was 5% per annum and therefore the sum required in six years time would be £100,000 $(1.05)^6 = £134,010$. The tax free rate of return on pension investments was assumed to be 9%. Let us now assume that the annual contributions are increased at the 6.5% rate of growth of national average earnings rather than the 5% rate of inflation. How much would Mr Stone need to invest initially to achieve the target value of his fund in 6 years time? Let us set out the sums involved, assuming the initial investment of £X is made immediately.
Table 7
Mr Stone’s Required Saving And Investment Returns

<table>
<thead>
<tr>
<th>NUMBER OF YEARS TO INVESTMENT</th>
<th>AMOUNT INVESTED</th>
<th>INVESTMENT GROWTH</th>
<th>FUTURE VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>£x</td>
<td>(1.09)^6</td>
<td>£x (1.09)^6</td>
</tr>
<tr>
<td>1</td>
<td>£x(1.065)</td>
<td>(1.09)^5</td>
<td>£x(1.065)(1.09)^5</td>
</tr>
<tr>
<td>2</td>
<td>£x(1.065)^2</td>
<td>(1.09)^4</td>
<td>£x(1.065)^2(1.09)^4</td>
</tr>
<tr>
<td>3</td>
<td>£x(1.065)^3</td>
<td>(1.09)^3</td>
<td>£x(1.065)^3(1.09)^3</td>
</tr>
<tr>
<td>4</td>
<td>£x(1.065)^4</td>
<td>(1.09)^2</td>
<td>£x(1.065)^4(1.09)^2</td>
</tr>
<tr>
<td>5</td>
<td>£x(1.065)^5</td>
<td>(1.09)</td>
<td>£x(1.065)^5(1.09)</td>
</tr>
<tr>
<td></td>
<td>TOTAL</td>
<td></td>
<td>TOTAL £134,010</td>
</tr>
</tbody>
</table>

We can solve for the value of £X using the method first explained in sections 20 and 22. Divide the future value by one plus the annual rate of increase in the amount invested raised to the power of the number of years of investment growth available for the first investment. This means dividing each entry in the final column by (1.065)^6 to obtain:

$$\frac{X(1.09)^6 + X(1.09)^5 + \ldots + X(1.09)^2 + X(1.09)}{(1.065)^6 + (1.065)^5 + \ldots + (1.065)^2 + 1} = \frac{134,010}{91,841.63} = 91,841.63$$

This is a regular savings calculation with an investment rate of return equal to:

$$\frac{1.09}{1.065} - 1 \times 100 = 2.35\%$$

We can solve for the value of £X using our calculator as follows:

1. Let the number of payments per year be equal to 1.
2. Check that BEGIN is displayed as the first payment is at the beginning of each year.
3. Enter the following known values:
   - I/YR = 2.35
   - FV = 91,841.6
   - PV = 3
   - N = 0
   - 6.00
4. Press PMT = -14,101.92

Thus the initial investment Mr Stone must make is £14,101.92, and this should increase by 6.5% per annum, the rate of increase in earnings. By the end of the sixth year, the value of the fund...
should equal the required £134,010 if investment returns are 9% per annum.

Table 8 below sets out the cash flows for each year and shows our calculation to be correct.

**Table 8: Mr Stone: Proposed Capital Accumulation**

<table>
<thead>
<tr>
<th>NO. OF YEARS TO INVESTMENT</th>
<th>CONTRIBUTION AT START OF YEAR (£)</th>
<th>CAPITAL AT START OF YEAR (£)</th>
<th>GROWTH @ 9% (£)</th>
<th>CAPITAL AT END OF YEAR (£)</th>
<th>REAL FUND (£)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>14,101.92</td>
<td>14,101.92</td>
<td>1,269.17</td>
<td>15,371.09</td>
<td>14,639.14</td>
</tr>
<tr>
<td>1</td>
<td>15,018.54</td>
<td>30,389.63</td>
<td>2,735.07</td>
<td>33,124.70</td>
<td>30,045.08</td>
</tr>
<tr>
<td>2</td>
<td>15,994.75</td>
<td>49,119.45</td>
<td>4,420.75</td>
<td>53,540.20</td>
<td>46,250.04</td>
</tr>
<tr>
<td>3</td>
<td>17,034.41</td>
<td>70,574.61</td>
<td>6,351.71</td>
<td>76,926.32</td>
<td>63,287.47</td>
</tr>
<tr>
<td>4</td>
<td>18,141.65</td>
<td>95,067.97</td>
<td>8,556.12</td>
<td>103,624.09</td>
<td>81,192.19</td>
</tr>
<tr>
<td>5</td>
<td>19,320.85</td>
<td>122,944.94</td>
<td>11,065.04</td>
<td>134,009.98</td>
<td>100,000.31</td>
</tr>
</tbody>
</table>

**23. Interest And Regular Receipts**

Another situation frequently met by planners is where the interest rate, present and future values are known, leaving either the period of time or the payment to be found.

For example, your client has £50,000 invested in the building society paying a nominal 5% per annum net of tax. Interest is credited to the account monthly. The client wishes to draw out capital and accumulated interest monthly so that at the end of five years nothing is left in the account. How much could be withdrawn per month?

To find the answer, the following steps are required:

(1) Enter 12 payments per year

(2) Check that BEGIN is displayed

(3) Enter the remaining known values:

\[
\begin{align*}
PV &= -50,000 \\
FV &= 0.00 \\
I/YR &= 5.00 \\
SHIFT\times\text{P/YR} &= 60
\end{align*}
\]
(4) Press PMT = £939.65

Thus if £939.65 is withdrawn each month, and 5% nominal per annum is credited to the remaining balance, the value of the account will not be equal to zero until after the sixtieth withdrawal. Note that if £50,000 is divided into 60 equal payments, the sum payable is only £833.33

If the interest rate had been credited annually, the calculation of I/Yr would have been:

<table>
<thead>
<tr>
<th>ACTION</th>
<th>DISPLAY</th>
</tr>
</thead>
<tbody>
<tr>
<td>Enter: 5</td>
<td>5</td>
</tr>
<tr>
<td>Press: SHIFT EFF%</td>
<td>5.00</td>
</tr>
<tr>
<td>Press: SHIFT NOM%</td>
<td>4.89</td>
</tr>
</tbody>
</table>

With interest only credited annually in place of monthly, the monthly withdrawal must be reduced from £939.65 to £937.20.

24. Exercise

Mrs Todd has £27,500 invested in a building society account earning a nominal 4% per annum after tax which is credited annually. How much could she withdraw per month if the sum invested is to last seven years?

Answer: £373.76

How much could she withdraw per month if she wants the remaining balance in seven years to equal £1,000?

Answer: £363.44

For how many months could she afford to withdraw £450 per month?

Answer: 68.05 months (without the £1,000 left in the account at the end)

25. Escalating Receipts or Withdrawals

Let us return to your client who in Section 23 wanted to know how much could be withdrawn per month from a £50,000 building society deposit over 5 years so that at the end of the 5 years, the balance in the account would have been reduced to zero. In section 23, the monthly withdrawals were to be constant. What if the client wants the withdrawals to increase annually by say 4%? How much could be withdrawn in the first year? Let us assume that the rate of interest on the building society account is 5% in the example in section 23.
The calculation required is very similar to that in section 15 on escalating savings plans. This time, the known variables are PV, FV, I/Yr, N and the variable whose value is to be calculated as PMT. As before, the calculation involves 3 steps:

(1) Assume that the withdrawals are annual and that the first withdrawal is made at the beginning of the first year. Calculate the value of this first annual withdrawal.

(2) Assume that the withdrawals are annual and that the first withdrawal is made at the end of the first year. Calculate the value of this first annual withdrawal.

(3) Calculate the average of (1) and (2) and divide by 12 to obtain the withdrawals that could be made in the first year if spread evenly at 12 monthly intervals. The withdrawals in each of the following years would be increased by 4%.

25.1 Calculation of annual withdrawal if made at the start of the year

(1) Enter one payment per year.
(2) Check that BEGIN is displayed.
(3) Enter the required future value of zero
(4) Calculate the annual interest rate to be used. This is one plus the annual rate of interest divided by one plus the rate of increase in the annual withdrawal; subtract one, and multiply by one hundred to convert into a percentage.

\[
\frac{1.05}{1.04} - 1 \times 100 = 0.96\%
\]

(5) Press the I/Yr button to enter this value [NB: since the payments are annual, the effective rate and the nominal rate are the same]

In this example, the annual interest rate is 5% and the annual rate of increase in withdrawal is 4%. Thus the interest rate per year to be used in the calculation is:

\[
1.05 - 1 \times 100 = 0.96\%
\]

(6) Enter the number of years of withdrawals. In this example, N will be equal to 5.
(7) Enter the initial investment as the present value. In this example, the PV is equal to \(-50,000\)
(8) Calculate the annual withdrawal by pressing PMT. In this example, £10,192.30 could be withdrawn immediately, and an amount 4% larger each year until the commencement of the fifth year, at which time the remaining balance in the account after the fifth annual withdrawal would be zero.
26. Calculation of Annual Withdrawal if Made at the End of the Year

Since the first withdrawal has been delayed until the end of the first year, the money available in the account would have accrued one year’s interest before withdrawals commence. To obtain the amount that could therefore be withdrawn at the end of the first year, we only need to multiply the result of the first calculation by one plus the annual rate of interest paid by the building society.

In this example:

\[ \£10,192.30 \times 1.05 = \£10,701.92 \]

27. Calculation of the monthly Withdrawal in the First Year

The amount which can be withdrawn per month in the first year is found by taking the average of the possible annual withdrawals, and dividing by 12.

\[
\frac{\£10,192.30 + \£10,701.92}{2} \div 12 = \£870.59
\]

Thus your client can withdraw £870.59 per month for the first year and on each anniversary increase the withdrawal by 4% and the account would not be empty until the sixtieth withdrawal had been made.

A similar calculation is required if the client wants a specific payment each month which increases each year, but the amount that needs to be invested is unknown. Let us use the same figures as before. Your client wishes to invest in a building society a large enough sum so that £870.59 can be withdrawn each month in the first year. On each anniversary the monthly sum withdrawn is to increase by 4% and the withdrawals are to be made for five years. At the end of the 5 years, the balance of the account is to be zero. The rate of interest credited annually by the building society is 5% net of tax. What is the initial sum to be invested?

Finding the amount your client would need to invest involves 3 stages similar to the previous calculation:

(1) Calculate the sum that would need to be invested in the building society if the withdrawals were annual and the first withdrawal is immediate.

(2) Calculate the sum that would need to be invested in the building society if the withdrawals were annual and the first withdrawal is at the end of the first year.

(3) The sum that needs to be invested to provide the income required is approximately equal to the average of (1) and (2)
27.1 Calculation of initial investment if the first annual withdrawal is immediate.

(1) Enter one payment per year.

(2) Check that BEGIN is displayed.

(3) Enter the required future value of zero.

(4) Calculate the annual interest rate to be used. This is one plus the annual rate of interest divided by one plus the rate of increase in the annual withdrawal; subtract one, and multiply by 100 to convert into a percentage.

(5) Press the I/Yr button to enter this value.

   In this example, the annual interest rate is 5 per cent and the annual rate of increase in withdrawal is 4%. Thus the interest rate per year to be used in the calculation is:

   \[
   \frac{1.05 - 1}{1.04} \times 100 = 0.96% \]

(6) The required payment would be:

   \[12 \times £870.59 = £10,447.08\]

(7) Calculate the required initial investment by pressing the PV button.

   In this example, PV = £51,249.87

27.2 Calculation of initial investment if the first annual withdrawal is at the end of the first year.

Since the withdrawal is not until the end of the first year, there is 12 months of interest that will accrue before the withdrawal takes place. The initial investment need not therefore be as great as in the first situation where the first withdrawal was immediate. The initial investment in the first case can be reduced to the level where the sum invested together with the first year’s interest is sufficient to provide an immediate withdrawal of £10,447.08, and four further annual withdrawals, each 4 per cent greater than the previous withdrawal.

The amount that needs to be invested is found by dividing the answer to (27.1) above by one plus the rate of interest paid by the building society.
In this example:

\[\£51,249.87 = \£48,809.40 \]

\[\frac{1.05}{27.3} \]

### 27.3 Calculation of the required initial investment

The initial investment required to produce the required withdrawals is approximately equal to the average of the values found in (27.1) and (27.22) above. In this example:

\[(\£51,249.87 + \£48,809.40) \div 2 = \£50,029.64\]

We know this to be approximately correct as the previous calculation in this section produced the required income from an initial investment of £50,000.

### 28. Practice Examples

(1) Mr Underwood seeks an income from a high interest bank account of £1,800 per month for the next ten years increasing annually at 6%. The rate of interest that he anticipates is 8.5%. What would be the required size of the initial deposit if the balance was to be run down to zero by the last withdrawal?

Answer: £187,290.48

(2) Mrs Williams comes to you with £80,000 and wants to know what approximate level of initial monthly income this might provide increasing at 5% per annum for 7 years if invested in a building society account paying a rate of interest of 7.5% and if the balance is to be run down to zero by the last withdrawal?

Answer: £1,059.19 per month

### 29. Use in Every Day Financial Planning and Advising

A financial calculator (or computer package) will usually provide the following five buttons to allow values to be input or calculated for each:

- The number of time periods : N
- The rate of interest : I/YR
- The present value : PV
- The payment : PMT
- The future value : FV
The power of these tools of analysis lies in the fact that if the values of any four of these variables are known, then the calculator can work out for us the value of the fifth variable. Thus, in essence, we have the tools to analyse five alternative scenarios:

<table>
<thead>
<tr>
<th>Variable</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
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</thead>
<tbody>
<tr>
<td>N</td>
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<td>FV</td>
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</tbody>
</table>

The process of personal Financial Planning for the professional planner frequently involves the following stages of analysis:

1. Recognition of the true nature of the questions that need to be answered given the client’s circumstances.
2. Formulate the questions within a model that is solvable.
3. Calculate the numerical solution.

To demonstrate this, there are examples of each of the five situations depicted in the table. It is likely that you will have experienced the same or many similar situations yourselves.

29.1 Scenario One: N – The Number of Years

A client had a power of attorney to look after the finances of a relative in his eighties who was in a nursing home with Alzheimer’s Disease. The income from the investments had just become insufficient to meet the fees of the nursing home. Another relative, who also shared the power of attorney, was arguing for the purchase of an annuity in the hope of ensuring that the income would be sufficient to meet the fees in future. This started a family row between all the potential beneficiaries who felt that an annuity would remove any possibility of their receiving a share of the estate on eventual death.

In spite of making a very conservative assumption about the future rate of interest, it can be demonstrated to this client, after using a time value of money calculation, that if income and capital were drawn each year to meet the nursing home fees, the capital would last until the client was 108, even after allowing for reasonable annual increases in the nursing home fees.

Using the calculator or a spread sheet allows “what if” questions to be answered quite quickly. Different scenarios can be examined assuming different rates of interest on the investments and different rates of increase in the payments. The Present Value is set to the capital available, the Future Value of the capital is set to zero, and the model enables you to find the value of N, the number of years that the capital and interest can sustain the specified payment withdrawals.
29.2 Scenario Two: I/YR - The Rate of Interest

This calculation applies to where you know the capital values now and at some point in the future, the amount of withdrawals and their frequency (if any) and you need to know the rate of return on investments that has occurred to give the future capital value.

Examples of this situation may arise where you are advising a client on whether to purchase added years in a public sector pension scheme or whether to use an FSAVC.

There are many aspects of this problem that you would discuss with your client, such as:-

1. The purchase of added years involves a predictable addition to the pension in retirement which is not dependent upon stock market performance. The benefit of an FSAVC does depend upon both stock market performance and annuity rates.

2. The purchase of added years includes an enhancement to death in service benefits dependent upon the number of years purchased. An FSAVC can provide increased life cover, and the client can select the amount, within Inland Revenue limits.

3. The purchase of added years is especially valuable if the years are purchased when the salary is relatively low, and then as a result of promotion, the pension is based on a much higher final salary.

Almost inevitably, a client is going to ask which of the alternatives will provide the larger pension. The answer to this question will probably be a major factor in their decision. Given that the size of an FSAVC pension depends upon future investment performance and annuity rates, this question is impossible to answer with any certainty.

One way round this dilemma is to estimate the required rate of investment return that would be necessary to provide a better pension than that provided by the purchase of added years. How do we do that?

Let’s assume that your client has a salary of £30,000 and 12 years to go to retirement at age 60.

We have to make an assumption as to what the average rate of increase in salary will be over the next 12 years in order to estimate the final salary at retirement. The assumed rate of salary increase can be varied to see what effect it has on the final outcome.

If we assume that the salary will increase at 3% per annum, your client’s final salary in 12 year’s time at age 60 will be £42,773.

If your client decides to purchase five added years by having 9% of gross income deducted every month for the next 12 years, the benefits on retirement at age 60, given that it is a public sector scheme, will be :-

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An increase in pension of $\frac{5}{80} \times £42,773 = £2,673$

An increase in tax free lump sum of $\frac{15}{80} \times £42,773 = £8,020$

In order to work out what the value of these benefits will be in 12 years’ time, we need to place a cash value on the index-linked pension of £2,673.

Let us make an assumption about the cost of an index linked pension annuity for a male aged 60, a wife aged two years younger which continues at 50% in the event of the client predeceasing his wife. Let us assume that each £1,000 per annum of such an indexed linked pension annuity would cost £16,000.

This means that the value of the index linked pension at age 60 of £2,673 is £16,000 x 2.673 = £42,768.

Thus, to match the benefits of added pension and added lump sum from the purchase of the five added years of service at age 60 would cost £42,768 + £8,020 = £50,788.

To obtain these benefits valued at £50,788 in 12 years’ time, your client would have to pay 9% of his salary each month. This would be £225 per month, increasing at 3% every year.

The next step is to use a financial calculator to work out what rate of investment return would be required by the client investing £225 per month, increasing at 3% per annum, in order to accumulate a fund equal to £50,788. The required rate of return is 4.90%.

There are two adjustments that need to be made to these figures. Firstly, the cost of providing equivalent death in service benefits to those obtained from purchasing added years service needs to be subtracted from the monthly premium that is invested, thereby raising the required rate of return above 4.90%.

Secondly, the calculation has ignored the effect of charges upon investment performance. Let us assume that the net effect of the additional life cover and the administrative charges is to raise the required rate of return to 7.5%.

You can now say to your client that to provide an equivalent pension to that provided by the purchase of the five added years, the FSAVC must achieve an investment rate of return in excess of 7.5% per annum. You can then discuss with him the likelihood of achieving this in an economic environment where public sector pay is rising by 3% per annum.
29.3 Scenario Three: PV - The Present Value

This type of calculation is very common. A question frequently asked by clients is, “How much do I need to invest now in order to be able to fund my children or grandchildren through school or university?” If the saving is to take the form of a one off up front lump sum, rather than a regular saving scheme, finding the Present Value is the type of calculation involved.

We assume a rate of growth of investments, assume future contributions will be nil, we know the length of time until the capital is required, we know the Future Value of the amount of capital required, and the calculator provides us with the answer that we need - the amount of capital that needs to be invested now.

It is often the case that this type of problem needs two calculations of Present Value, not one. It is unlikely that the client actually knows the Future Value or the amount of capital that will be required to fund the child or grandchild through, say, university.

The first stage of the analysis then becomes a calculation of the capital required for this purpose. Start by working out what would be required this year, if the child were to commence university now. We assume a rate of inflation of student costs, and calculate what the requirement is likely to be in the actual year of commencement at university. This becomes the annual payment. Remember that this too must be increased by the rate of inflation for the student’s second and final years at university.

The Future Value of capital left after three years of support will be zero. We assume a cautiously low rate of investment return on the balance of capital not yet given to the student. We know the payments will cover three years. Thus we can use the calculator to find for us the Present Value or amount of capital required at the commencement of university to see the student through the three years of study.

29.4 Scenario Four: PMT - Payment

Probably one of the most common uses of this calculation is in retirement planning. We start by asking our clients how much income they would like in retirement to sustain the lifestyle they desire.

We then isolate how much of that income will be required from the pension, and how much we are to assume will be funded from other sources.

Once we have the required pension, we can apply cautious annuity rates in order to find the amount of capital that will need to be in the pension plan at retirement age to provide the required level of pension.

The next step is to find out the existing value of the pension plan and to project that forward at an assumed rate of investment growth to the desired retirement age. By subtracting this anticipated
pension fund due to previous contributions from the desired pension fund at retirement, we can calculate the shortfall.

This shortfall is the Future Value, the capital that has to be accumulated. We know the number of years which are available to accumulate it, that the Present Value or existing fund to provide this shortfall is zero, and the assumed rate of return on investment. The calculator can now provide us with the value of the payment or regular premium required to provide the extra capital required. This regular payment will assume that future contributions to the pension plan are level or constant. However, only a small and easy adjustment to the calculation is required to find the first year’s premiums required assuming that the premiums are increased each year at a constant rate.

29.5 Scenario Five : Future Value

A client is about to retire from the Royal Navy at the age of 33. As a skilled electronics engineer, he has no real concern about being able to find alternative employment. The Royal Navy is to commence his pension as soon as he leaves, but they had provided him with a choice:-

Option One - A lump sum of £48,500 and an annual pension of £4,900.

Option Two - A lump sum of £65,000 and an annual pension of £3,000 which at age 55 would be increased to equal the pension that he would have received had he chosen the first option.

The pension would be index linked to the RPI, and a spouse’s pension of two thirds of the main pension would also be provided.

There were many factors to be taken into consideration by this client, not least whether he had an immediate need of a higher income or whether he had need of the lump sum to move house to obtain a new job. However, concentrating on the time value of money aspect of the problem, the key question could be identified as :-

“Is it better to have an extra £16,500 lump sum now, or an extra index linked pension starting at £1,900 per annum for the next 21 years until age 55? Which alternative will provide the client with a larger amount of capital at 55 with which to provide his income in retirement?”

The interesting thing about this problem is that the higher the rate of inflation, relative to the rate of investment return, the more attractive the index linked pension becomes as a source or regular income to invest to age 55.

The way to tackle this problem is by scenario analysis. For example, we could calculate the effect of a high investment return, low inflation scenario and compare it with the effect of a low investment return, low inflation scenario.
If we assume a 12% rate of investment return and a 3% rate of inflation:

- The lump sum of £16,500 grows to £178,263 by age 55
- Investment of the index linked pension of £1,900 pa provides capital of £200,137 at age 55
- If inflation drops to 0%, the extra pension provides only £164,561 - at rates of inflation greater than 3%, the advantage of the index linked pension grows dramatically.

If we assume a 5% rate of investment return and a 0% rate of inflation:

- The lump sum of £16,500 grows to £45,968 by age 55
- Investment of the index linked pension of £1,900 per annum provides capital of £71,260 at age 55

Hence the choice of the additional pension is most likely to provide a higher income in retirement provided it is saved and invested.